Frequency-Domain Calculation of Floquet Magnonic Band Structures

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Abstract: Here we developed a new approach to calculate the band structures of spin waves in waveguides that change periodically called magnonic crystals, by applying floquet boundary conditions to the frequency domain simulations. We optimized the analysis of width-modulated magnonic crystals and discovered the band-related edge states in skyrmion crystals. Our work may accelerate the research of topologically nontrivial phases in magnetic systems.

Introduction

The dynamics of spin waves in magnonic crystals can be illustrated by the Landau-Lifshitz-Gilbert (LLG) equation, where the effective field includes external magnetic field, exchange interaction, anisotropy, Dzyaloshinskii-Moriya interaction (DMI) etc. Micromagnetic simulation is performed by COMSOL Multiphysics (Finite Elements Method).

$$\frac{\partial \boldsymbol{m}}{\partial t} = -\gamma \boldsymbol{m} \times \boldsymbol{H}_{\text{eff}} + \alpha \boldsymbol{m} \times \frac{\partial \boldsymbol{m}}{\partial t}$$

$$\boldsymbol{H}_{\text{eff}} = \boldsymbol{H}_{\text{ext}} + A\nabla^2 \boldsymbol{m} + K m_z \hat{e}_z + D[(\nabla \cdot \boldsymbol{m})\hat{e}_z - \nabla m_z]$$

Frequency-Domain Simulation of LLG

Hypotheses of linearization:

$$\boldsymbol{M} = M_{\text{S}}(\boldsymbol{m}_0 + \delta \boldsymbol{m}\, \mathrm{e}^{\mathrm{i}\omega t})$$

$$\boldsymbol{H}_{\text{eff}} = \boldsymbol{h}_{0\text{eff}} + \delta \boldsymbol{h}_{\text{eff}} \mathrm{e}^{\mathrm{i}\omega t}$$

Floquet Boundary Condition:

$$\delta \boldsymbol{m}(\boldsymbol{r} + \boldsymbol{R}) = \delta \boldsymbol{m}(\boldsymbol{r}) \cdot \mathrm{e}^{-\mathrm{i}\boldsymbol{k}_{\text{F}} \cdot \boldsymbol{R}}$$

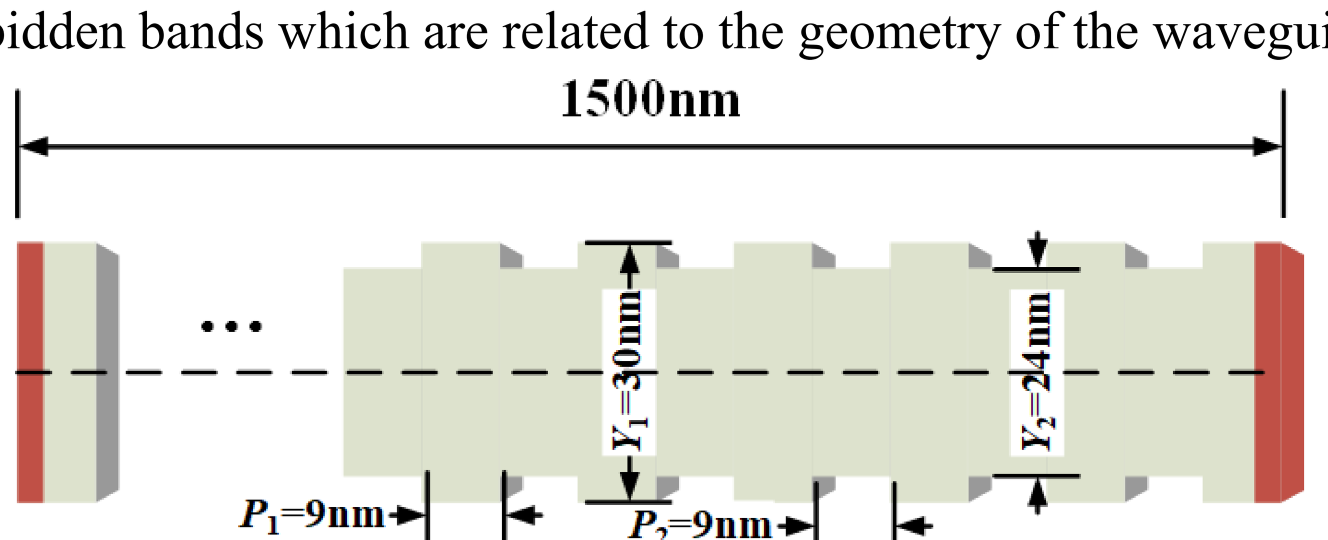
LLG equation in frequency domain after Fourier transform:

$$\mathrm{i}\omega \delta \boldsymbol{m} = -\gamma \boldsymbol{m}_0 \times \delta \boldsymbol{h}_{\text{eff}} - \gamma \delta \boldsymbol{m} \times \boldsymbol{h}_{0\text{eff}} + \mathrm{i}\omega \alpha \boldsymbol{m} \times \delta \boldsymbol{m}$$

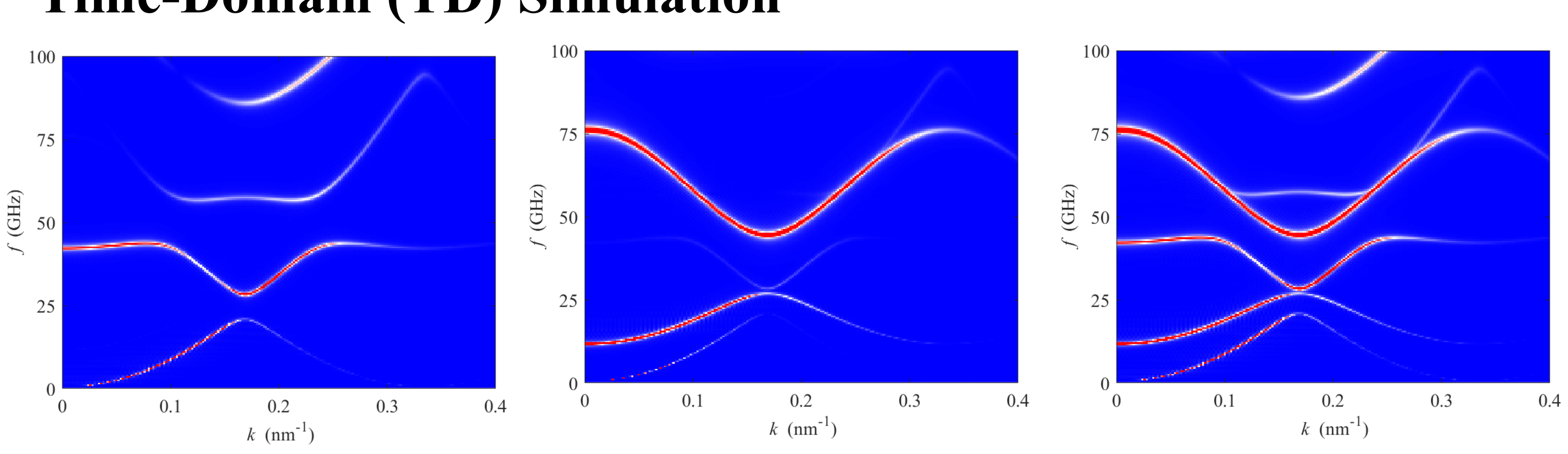
The calculation of floquet magnonic band structures is done by the method of eigenfrequency analysis.

Width-Modulated Magnonic Crystals (WMMC)

Magnonic crystals made up of periodically modulated widths could change the propagation of incident spin waves and form magnonic forbidden bands which are related to the geometry of the waveguides.



Time-Domain (TD) Simulation

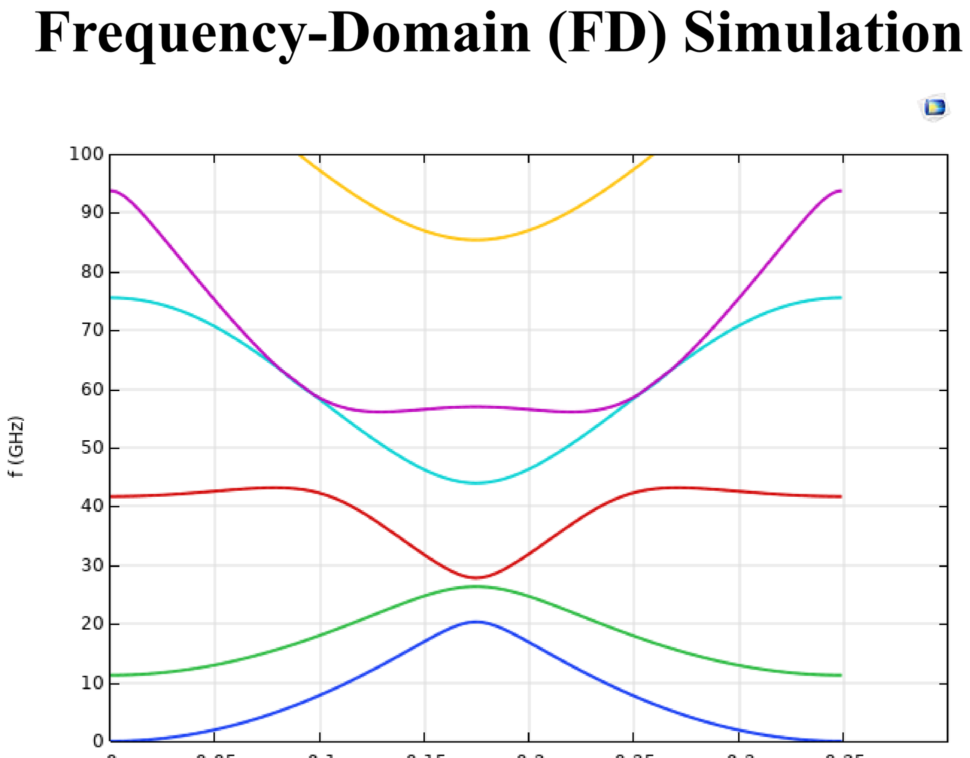


Even Term *Odd Term* *Mixed Term*

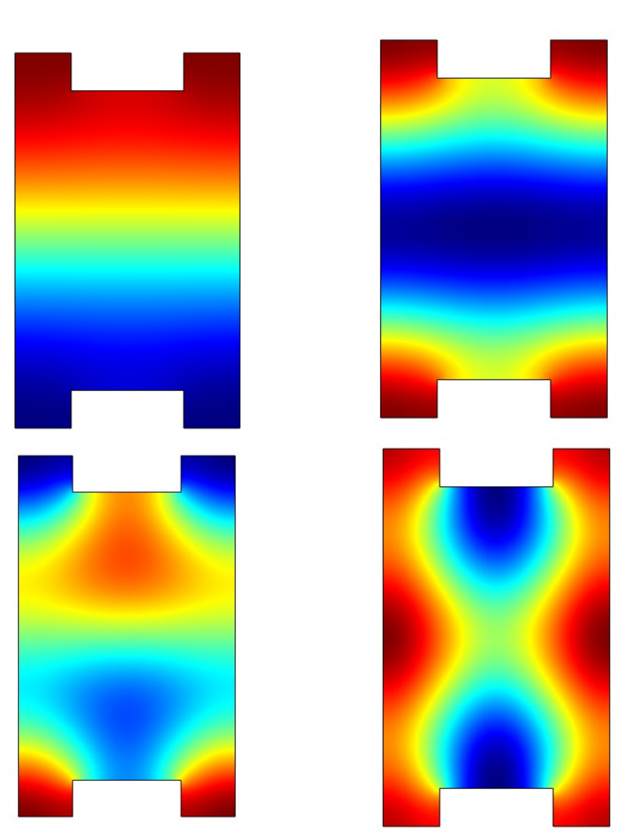
$$H_{y\,\text{ext}} = H_0 \frac{\sin(2\pi f_c t)}{2\pi f_c t} \cdot \left[\xi \cos\left(\frac{\pi y}{d}\right) + \zeta \sin\left(\frac{\pi y}{d}\right)\right]$$

2D-fast Fourier transform (FFT) is required to get the dispersion relations. The parities of spin waves in TD simulation are depended on the parities of external excitation magnetic fields.

Frequency-Domain (FD) Simulation



Floquet Magnonic Bands

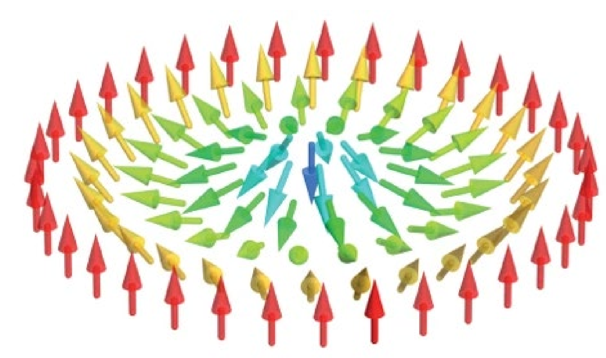


Spin Wave Excited States

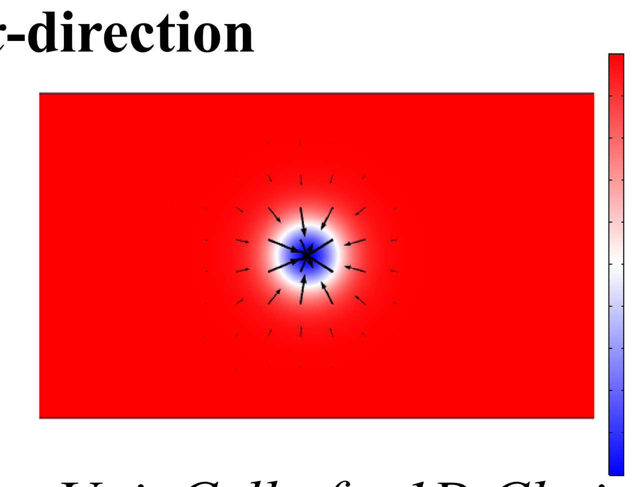
The Eigenfrequencies and Eigenmodes of spin waves found in FD simulation are composed of both parities. No more FFTs are needed!

Skyrmion Crystals (SkX)

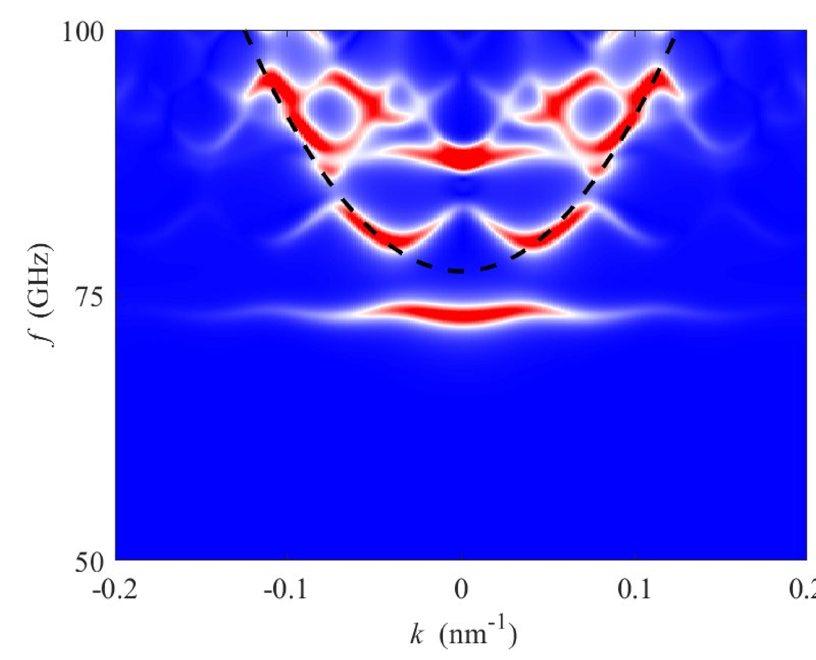
Infinite 1D Skyrmion Chain in *x*-direction



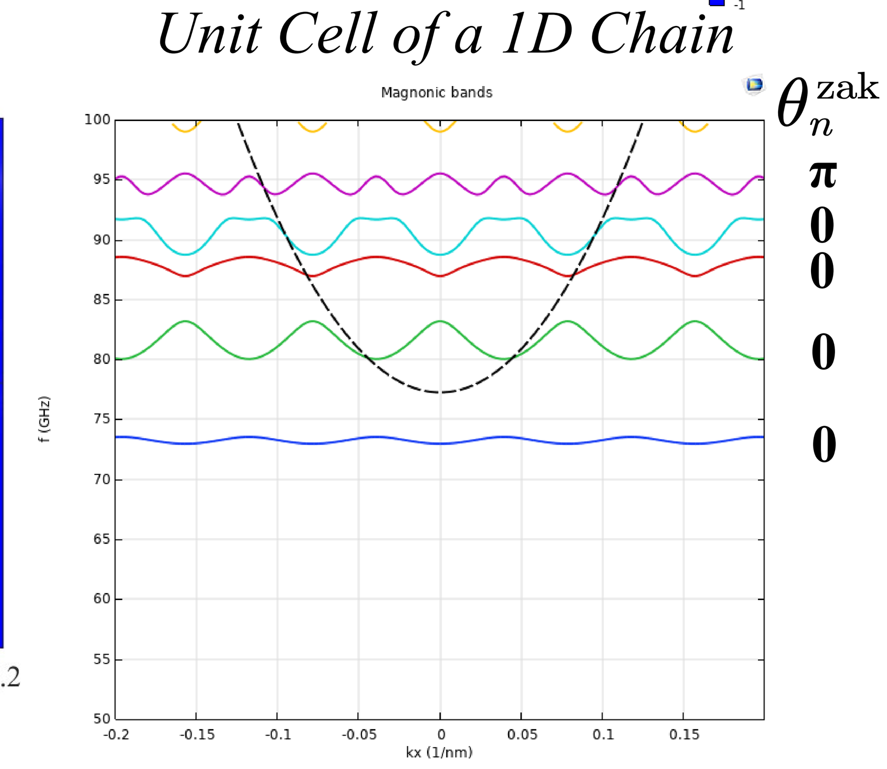
Neel Type Skyrmion



Unit Cell of a 1D Chain



TD Band Structure



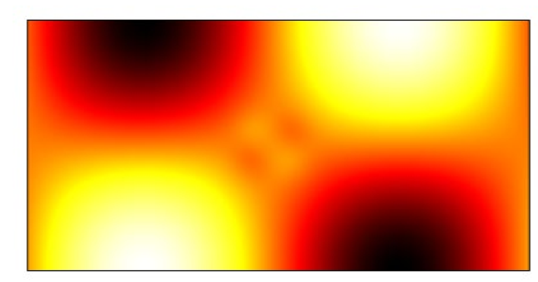
FD Band Structure

The nontrivial Zak Phase (Berry Phase) on the 5th band results from the reversal of spin wave parity in *y*-direction between $k = 0$ and $k = \pi/a$.

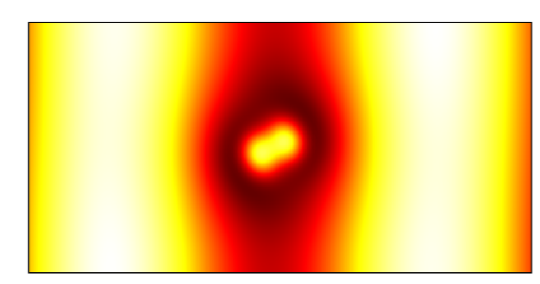
Zak Phase

$$\theta_n^{\text{zak}} = \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \left\{ \mathrm{i} \int_{\text{unit cell}} u^*(\boldsymbol{r}) \frac{\partial u(\boldsymbol{r})}{\partial k} \mathrm{d}\boldsymbol{r} \right\} \mathrm{d}k$$

$$u(\boldsymbol{r}) = \frac{\delta m_x(\boldsymbol{r}) \mathrm{e}^{\mathrm{i}kx}}{\sqrt{\int_{\text{unit cell}} |\delta m_x(\boldsymbol{r})|^2 \mathrm{d}\boldsymbol{r}}}$$

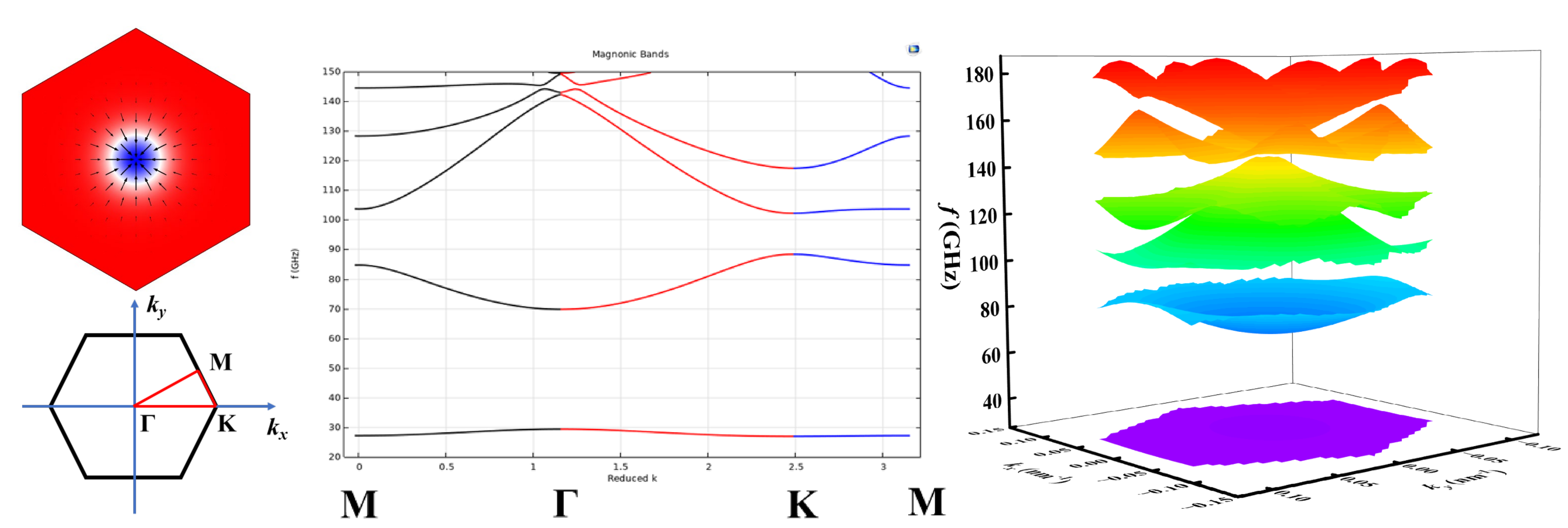


$k = 0$

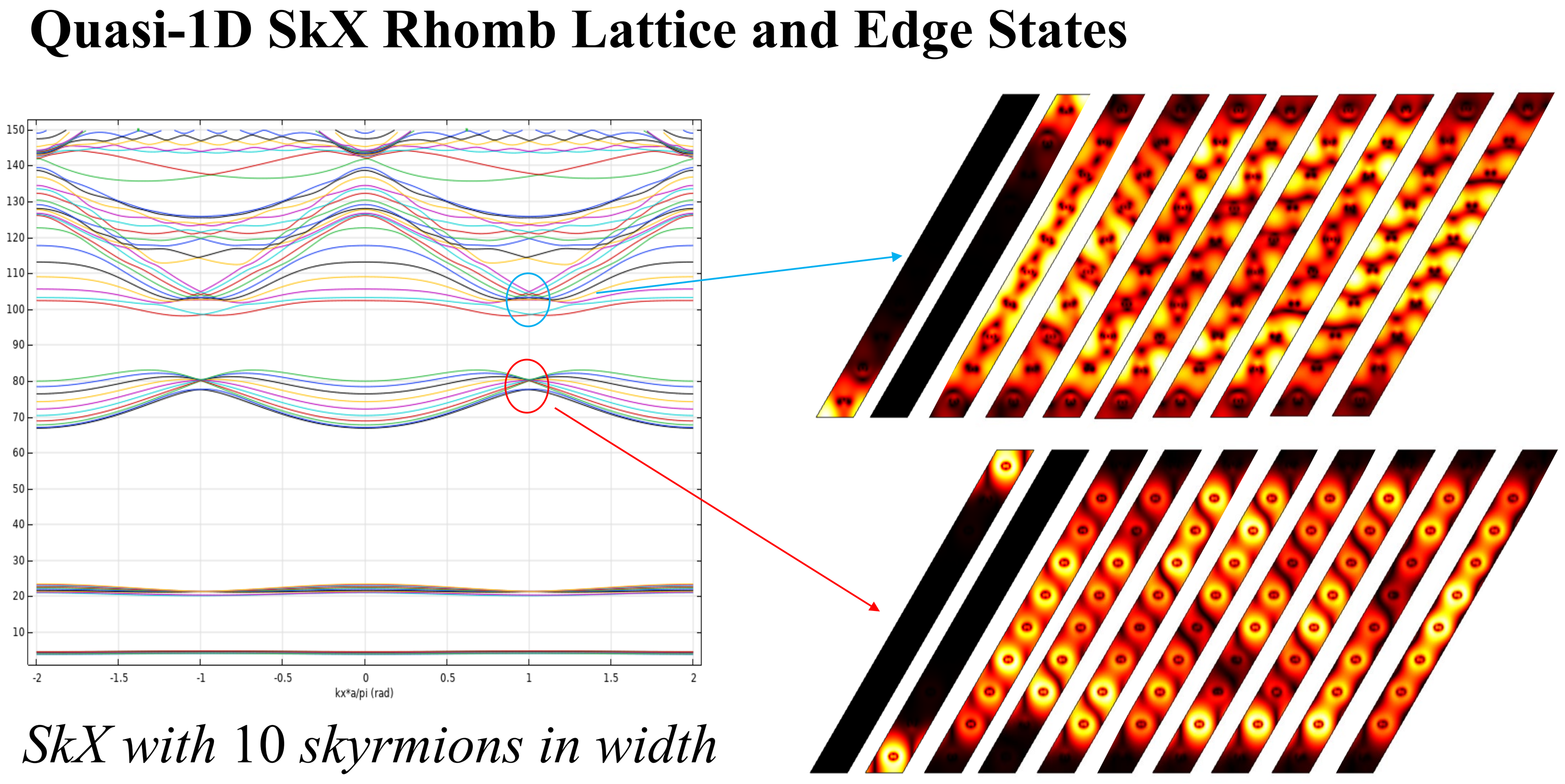


$k = \pi/a$

Infinite 2D SkX Rhomb Lattice



Quasi-1D SkX Rhomb Lattice and Edge States



SkX with 10 skyrmions in width

There are 2 bands of each energy level crossing on the boundaries of Brillouin zones, which have smaller frequencies and correspond to 2 edge states of upper and lower boundaries respectively, while other 8 bands of each level are all bulk modes.

Reference

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